Topic 05 – Algebra (Solutions)

Q1, Jan 2006, Q2

(i) Simplify $(3x+1)^2 - 2(2x-3)^2$. [3]

(ii) Find the coefficient of x^3 in the expansion of

$$(2x^3 - 3x^2 + 4x - 3)(x^2 - 2x + 1).$$ [2]

i/ $(3x+1)(3x+1) - 2(2x-3)(2x-3)$

$= (9x^2 + 3x + 3x + 1) - 2(4x^2 - 6x - 6x + 9)$

$= 9x^2 + 6x + 1 - 2(4x^2 - 12x + 9)$

$= 9x^2 + 6x + 1 - 8x^2 + 24x - 18$

$= x^2 + 30x - 17$

ii/ $(2x^3 - 3x^2 + 4x - 3)(x^2 - 2x + 1)$. $4x^3 + 6x^3 + 2x^3 = 12x^3$

$4x^3$, $6x^3$, $2x^3$

$\therefore$ Coefficient = 12

Q2, Jun 2006, Q4i

By expanding the brackets, show that

$$(x-4)(x-3)(x+1) = x^3 - 6x^2 + 5x + 12.$$ [3]

$(x-4)(x-3)(x+1) = (x^2 - 7x + 12)(x+1)$

$= x^3 + x^2 - 7x^2 - 7x + 12x + 12$

$= x^3 - 6x^2 + 5x + 12$

Q3, (Jun 2007, Q1)

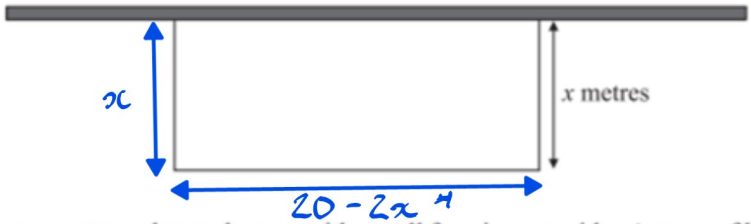
Simplify $(2x+5)^2 - (x-3)^2$, giving your answer in the form $ax^2 + bx + c$. [3]

$(2x+5)(2x+5) - (x-3)(x-3)$

$= 4x^2 + 20x + 25 - (x^2 - 6x + 9)$

$= 4x^2 + 20x + 25 - x^2 + 6x - 9 = 3x^2 + 26x + 16$

Q4, (Jun 2007, Q5i)



The diagram shows a rectangular enclosure, with a wall forming one side. A rope, of length 20 metres, is used to form the remaining three sides. The width of the enclosure is x metres.

(i) Show that the enclosed area, A m^2, is given by

$$A = 20x - 2x^2.$$ [2]

* (Since all three sides sum to 20)

∴ Area $= x(20 - 2x) = 20x - 2x^2$

Q5, (Jun 2008, Q6i)

Expand and simplify $(x-5)(x+2)(x+5)$. (Difference of two squares) [3]

$(x+5)(x-5)(x+2) = (x^2-25)(x+2) = x^3 + 2x^2 - 25x - 50$

Q6, (Jan 2012, Q3)

Given that

$$5x^2 + px - 8 = q(x-1)^2 + r$$

for all values of x, find the values of the constants p, q and r. [4]

$5x^2 + px - 8 = q(x^2 - 2x + 1) + r$

$\Rightarrow 5x^2 + px - 8 = qx^2 - 2qx + q + r$

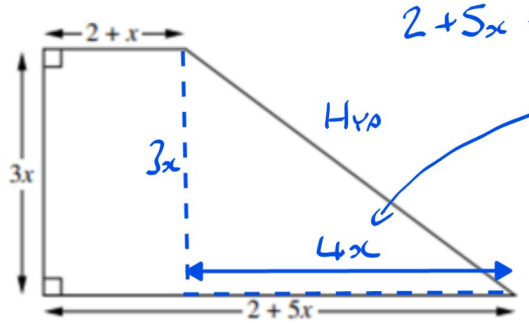
Comparing coeffts, $q = 5$, $p = -2(5) = -10$

$5 + r = -8 \Rightarrow r = -13$

∴ $p = -10$, $q = 5$, $r = -13$

Q7, (Jan 2010, Q11i,ii)

A lawn is to be made in the shape shown below. The units are metres.



$2+5x-(2+x)=4x$

(i) The perimeter of the lawn is P m. Find P in terms of x. **[2]**

(ii) Show that the area, A m^2, of the lawn is given by $A = 9x^2 + 6x$. **[2]**

i/ $\text{Hyp} = \sqrt{(3x)^2 + (4x)^2} = \sqrt{25x^2} = 5x$

$\therefore P = (2+5x) + 3x + (2+x) + 5x$

$\Rightarrow \boxed{P = 14x + 4}$

ii/ Using $A = \frac{1}{2}(a+b)h$ (area of a trapezium)

$A = \frac{1}{2}(2+x+2+5x)(3x)$

$= \frac{1}{2}(4+6x)(3x) = (2+3x)(3x) = \boxed{6x+9x^2}$

Q8, (Jun 2010, Q4ii)

Expand $(x-2)^2(x+1)$, simplifying your answer. **[3]**

$(x^2-4x+4)(x+1) = x^3+x^2-4x^2-4x+4x+1$

$= x^3 - 3x^2 + 1$

Q9. (Jan 2011, Q2)

Given that

$$(x-p)(2x^2+9x+10) = (x^2-4)(2x+q)$$

for all values of x, find the constants p and q. **[3]**

LHS $= 2x^3 - 2px^2 + 9x^2 - 9px + 10x - 10p$

$= 2x^3 + (-2p+9)x^2 + (-9p+10)x - 10p$

RHS $= 2x^3 + qx^2 - 8x - 4q$

$\Rightarrow -2p + 9 = q$, $-9p + 10 = -8$, $-10p = -4q$

$\Rightarrow 9p = 18 \Rightarrow \boxed{p = 2}$

$\Rightarrow -2(2) + 9 = \boxed{q = 5}$

Q10, (Jun 2012, Q1)

Simplify $(x-5)(x^2+3)-(x+4)(x-1)$. [3]

$x^3 + 3x - 5x^2 - 15 - (x^2 + 3x - 4)$

$= \boxed{x^3 - 6x^2 - 11}$

Q11, (Jan 2013, Q5)

(i) Simplify $(x+4)(5x-3)-3(x-2)^2$. [3]

(ii) The coefficient of x^2 in the expansion of

$$(x+3)(x+k)(2x-5)$$

is -3. Find the value of the constant k. [3]

i/ $5x^2 - 3x + 20x - 12 - 3(x^2 - 4x + 4)$

$= 5x^2 + 17x - 12 - 3x^2 + 12x - 12$

$= \boxed{2x^2 + 29x - 24}$

ii/ $(x^2 + kx + 3x + k)(2x - 5)$

$= 2x^3 - 5x^2 + 2kx^2 - 5kx + 6x^2 - 15x + 2k - 5k$

Coefft of x^2 is $-5 + 2k + 6 = 1 + 2k$

$1 + 2k = -3 \Rightarrow 2k = -4 \Rightarrow \boxed{k = -2}$

Q12, (Jun 2016, Q1)

(i) Simplify $(2x-3)^2-2(3-x)^2$. **[2]**

(ii) Find the coefficient of x^3 in the expansion of $(3x^2-3x+4)(5-2x-x^3)$. **[2]**

i/ $(4x^2-12x+9)-2(9-6x+x^2)$

$= 4x^2-12x+9-18+12x-2x^2$

$= \boxed{2x^2-9}$

ii/ $(3x^2-3x+4)(5-2x-x^3)$. $\quad -6x^3-4x^3 = -10x^3$

$\therefore$ Coefft $= -10$